Table 1 Numerical comparison

d, in.	9.55567	6.37045	3.82227
A_s/td calculated	0.523249	0.523249	0.523249
$(t/d)^2(E/\bar{\sigma}_o)$ calculated	0.028168	0.063378	0.176051
Stress ratio:			
Based on Eqs. (6) and (12)	1.7742	1.5230	1.1672
Based on Eqs. (A1) and (A2)	1.7895	1.5353	1.1693
Ratio of results based on these two methods	0.9915	0.9920	0.9982

A_s/td, $(t/d)^2(E/\bar{\sigma}_o)$, and R/d. The last one is the number of stringers divided by 2π.

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"Power Law" Profiles in Thermally Stratified Shear Flows

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Nomenclature

c_p = Specific heat of air at constant pressure, cal/°C/gm
g = gravitational acceleration, cm/sec^2
H = heat flux in the vertical direction, cal/cm^2/sec
k = von Kármán constant
K_h = eddy thermal diffusivity, cm^2/sec
K_m = eddy viscosity, cm^2/sec
L = Monin-Obukhov length scale, $L' = (K_h/K_m)L = (Tu_*/gk)(\partial U/\partial z)/(\partial T/\partial z)$, cm
N = total number of data collected in a profile
R = dimensionless lapse rate, $(z/T_*)(\partial T/\partial z)$
Ri = $(g/T)(\partial T/\partial z)/(\partial U/\partial z)^2$, Richardson number
S = dimensionless wind shear, $(kz/u_*)(\partial U/\partial z)$
T = mean absolute temperature, °K
T_* = $-H/(c_p\rho k u_*)$, scaling temperature, °C
u_* = friction velocity, cm/sec
U = local mean velocity, cm/sec
z = height, cm
β, γ = arbitrary constants
ζ = z/L', dimensionless height
ρ = density of air, gm/cm^3
$(\ )_i$ = the variable at height z_i
$(\ )_m$ = mean value averaged over the profile
$(\ )_0$ = the variable at height z_0, an equivalent roughness height

1. Introduction

THE historically old "power law" has recently been revived by Pandolfo.[1] His profile has $-\frac{1}{6}$ power instead of $-\frac{1}{3}$ power profile of free convection tested by Taylor.[2] He claimed that $-\frac{1}{6}$ power law described the observed wind profiles quite accurately and that his model was more accurate to describe the wind profiles than Taylor's free convection model[2] or Swinbank's exponential model.[3] Pandolfo also compared his profile with KEYPS profile,[4] and concluded that these two models were comparable to each other pertaining to their accuracy of wind profiles description.

The mean velocity distribution in a neutral turbulent boundary layer is approximately given by Eq. (15) of Ref. 5 or Eq. (10) of Ref. 6. However, it is doubtful that the mean velocity distribution in thermally stratified shear flows will assume the same functional dependence on the height z as the neutral flow does. The power law model can indeed describe accurately the velocity profiles in thermally stratified shear flows. However, it is important to point out that the power should not be constant in all ranges of thermal stability, but rather dependent on the thermal stability as shown by Deacon[7] and should approach $-\frac{1}{3}$ profile as the flowfield approaches a free convection. The power profile is examined in this paper to show that this is the case for both laboratory and field data. Similarity between wind and temperature profiles is also studied.

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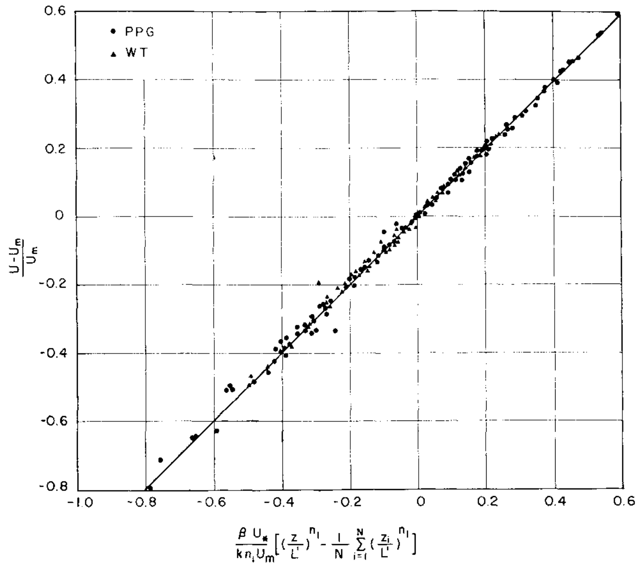

Fig. 1 Power law profile of the mean wind velocity.

2. Basic Equations

If the dimensionless wind shear and lapse rate are assumed to be proportional to ζ^n, then the mean wind velocity and the mean temperature will take the following forms:

$$U(z) - U_0(z_0) = (\beta u_*/kn)(\zeta^n - \zeta_0^n) \quad (1)$$

and

$$T(z) - T_0(z_0) = (\gamma T_*/n)(\zeta^n - \zeta_0^n) \quad (2)$$

The power n in the preceding equations should depend largely on the thermal stability of the flowfield. When data points are closely spaced and the respective velocity and temperature gradients in the vertical direction can be approximated by using finite difference techniques, the power n can be determined by the following equations:

$$\ln\frac{U_{i+1} - U_i}{z_{i+1} - z_i} = (n-1)\ln\frac{z_{i+1} + z_i}{2} + \ln\frac{\beta u_*}{kL'^n} \quad (3)$$

and

$$\ln\frac{T_{i+1} - T_i}{z_{i+1} - z_i} = (n-1)\ln\frac{z_{i+1} + z_i}{2} + \ln\frac{\gamma T_*}{L'^n} \quad (4)$$

where i varies from 1 to $N - 1$. If similarity between the velocity and the temperature profiles does not exist, then n will assume n_1 and n_2 in the preceding equations, respectively.

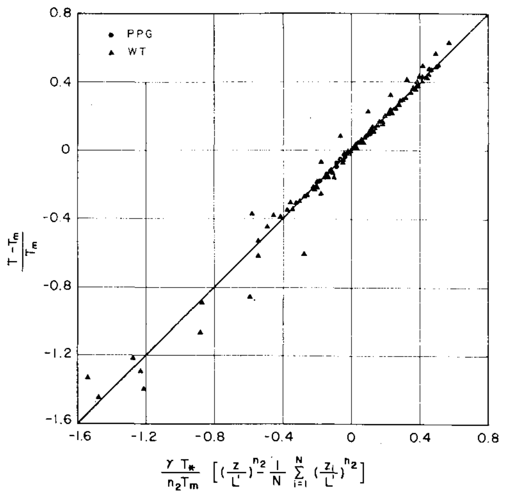

Fig. 2 Power law profile of the mean temperature.

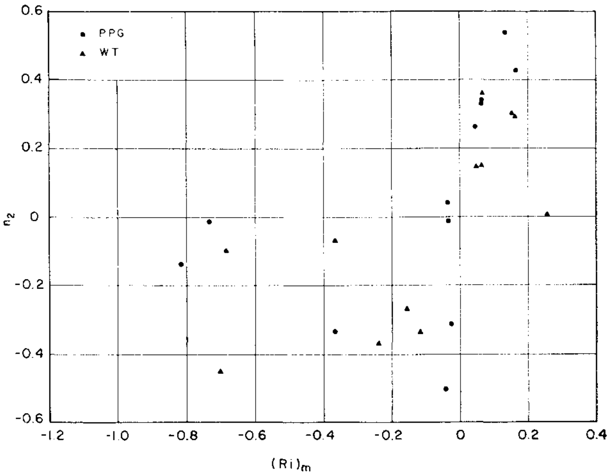

Fig. 4 Dependence of the power of mean temperature profiles upon Richardson number.

3. Experimental Results and Discussion

Mean wind velocity and temperature profiles in a wind-tunnel boundary layer over a horizontal flat plate that was cooled or heated depending on whether inversion or lapse condition was desired were measured and reported by Chuang and Cermak.[8] Data taken at equidistance of 0.7 cm. for height from 0.5 to 8.2 cm, were fitted to the log-linear model and were shown to have some scatter by the authors.[9] The same data were also fitted to the power law profiles by means of Eqs. (3) and (4) and the respective power of z, namely n_1 and n_2, for the velocity, and the temperature profiles were determined by the least squares method. The products $\beta u_*/kL'^{n_1}$ and $\gamma T_*/L'^{n_2}$ were evaluated in the same process. Equations (1) and (2) were rewritten as

$$\frac{U-U_m}{U_m}=\frac{\beta u_*}{kn_1U_m}\left[\left(\frac{z}{L'}\right)^{n_1}-\frac{1}{N}\sum_{i=1}^{N}\left(\frac{z_i}{L'}\right)^{n_1}\right]$$

and

$$\frac{T-T_m}{T_m}=\frac{\gamma T_*}{n_2T_m}\left[\left(\frac{z}{L'}\right)^{n_2}-\frac{1}{N}\sum_{i=1}^{N}\left(\frac{z_i}{L'}\right)^{n_2}\right]$$

Figures 1 and 2 show both the Project Prairie Grass[10] and the wind tunnel[9] data; the dimensionless velocity and temperature profiles, as shown respectively, have less degree of data scatter than those obtained by the other methods.

The power for wind and temperature profiles is shown, respectively, in Figs. 3 and 4. Although there is some degree of scatter, the functional dependence of n_1 and n_2 on the bulk thermal stability is revealed. Moreover, n_1 and n_2 have almost the same dependence on $(Ri)_m$, in accordance with the similarity hypothesis pertaining to the mean velocity and the mean temperature profiles. It should also be remembered that in free convection the power of a velocity profile is equal to $-\frac{1}{3}$. Results obtained by Deacon[7] are also shown approximately by a curve in Fig. 3.

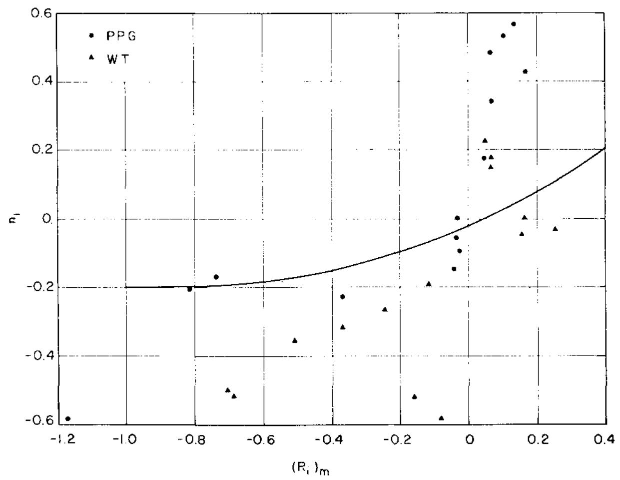

Fig. 3 Dependence of the power of mean velocity profiles upon Richardson number. The curve represents the results of Deacon's study.[7]

4. Conclusions

The power law model can yield a more accurate description of the wind and temperature profiles when the power n is given. However, unfortunately, the power of a profile is generally not known. The power was shown to be a function of the bulk Richardson number. More data are needed to determine this functional dependence. Nevertheless, this study shows that the power of wind and temperature profiles is not independent of the thermal stability.

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